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Early Universe in Scalar-Tensor Theory

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Abstract We consider the flat Robertson–Walker model in scalar-tensor theory proposed by Lau and Prokhovnik. In this model, the field equations are solved by using "gamma-law" form of equation of state $p = (\gamma - 1)\rho$, where the adiabatic parameter 'gamma' (γ) varies continuously as the universe expands. Our aim is to study how the adiabatic parameter γ should vary so that in the course of its evolution the universe goes through a transition from an inflationary to a radiation-dominated phase. A unified one parameter function of γ has been considered to describe the two early phases of evolution of universe. The solutions show the power-law expansion and cosmological constant is found to be positive and decreasing function of cosmic time. The solutions are compatible with the Dirac's large number hypothesis. The deceleration parameter has been presented in a unified manner in terms of scale factor, which describes the inflation of the model. The nature of singularity and the physical properties have been discussed in details.

Keywords Cosmology · Robertson–Walker models · Inflationary phase · Radiation-dominated phase

1 Introduction

Dirac [11] proposed a theory with variable G motivated by the numerology uncovered by Weyl, Eddington and Dirac himself. Dirac put the argument on a formal footing through his large numbers hypothesis (LNH), which states that any two of the very large numbers occurring in nature are connected by a simple mathematical relation in which the coefficient are of the order of unity. Since Dirac's work several attempts have been made to formulate

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a generalized field theory of gravitation in which G is a scalar function of coordinates and Einstein's theory of gravitation appears as a special case of the new theory. Jordan [15] attempted this by starting from a five-dimensional theory of relativity, where G is generalized to be a scalar function of the age of the universe and the covariant divergence of the energy–momentum tensor is non-zero. Dirac himself [12, 13] proposed the 'two metrics' theory where two unit systems are set up, namely the Einstein and the atomic units. Lau [16] proposed a theory by considering Einstein's field equations with a non-zero cosmological term Λ, with Λ and G assuming time-dependent forms in the field equations in order to satisfy the LNH without needing the 'two metrics' theory. Motivated by this conjecture and the LNH, Lau and Prokhovnik [17] generalized Lau's theory by formulating a new scalar-tensor theory in terms of an action principle. This is a theory with variable cosmological constant and gravitational constant but, in addition, it has a scalar field ψ. The theory developed is applied to a cosmological model compatible with the Dirac's Large Number Hypothesis. A time dependent scalar potential $\psi = \psi(t)$ is introduced such that $\Lambda = \Lambda(\psi)$ and $G = G(\psi)$ are coupled. This theory was further investigated by Beesham [3], Maharaj and Beesham [19] for the flat Robertson–Walker model. Maharaj and Naidoo [20] studied Robertson–Walker model in Lau–Prokhovnik's theory by imposing the constant deceleration parameter.

Nowadays, the parameter Λ is believed to correspond to the vacuum energy density of the quantum field [29], and it is thought that Λ was large during the early stages of the evolution of the universe. The status of the cosmological constant problem was reviewed by Weinberg [30]. It is worth noting that cosmological models with varying Λ have been the subjects of numerous papers in recent years. Bertolami [6, 7], Ratra and Peebles [23], Chen and Wu [10], Carvalho et al. [8] and Berman [4, 5] have worked out cosmologies based on time-dependent Λ. Abdel Rahman [1], Lima and Maia [18], Overduin and Cooperstock [21], Vishwakarma [28], Arbab [2] and Singh [25–27] studied cosmological models in a more phenomenological way by assuming some functional forms of Λ.

In cosmology, the evolution of universe is described by Einstein's equations of gravity together with an equation of state for the matter content. Usually the field equations are solved and analyzed separately for the different epochs, although some authors have given unified solutions. For instance, Israelit and Rosen [14] used a different equation of state in which the pressure varies continuously from $p = -\rho$ to its value during the radiation era ($p = \rho/3$) and then radiation to matter-dominated era ($p = 0$). Carvalho [9] presented a similar type of work by considering a model as it goes from an inflationary phase to a radiation-dominated era. He studied the model by using "gamma-law" equation of state but the adiabatic parameter 'gamma' varies with cosmic time as the universe expands.

In this paper we extend Carvalho's work in a scalar tensor theory proposed by Lau and Prokhovnik. We study the evolution of early universe for flat Robertson–Walker model as it goes from an inflationary phase to radiation-dominated era. In order to obtain the solutions for perfect fluid we consider the "gamma-law" equation of state $p = (\gamma - 1)\rho$,where the adiabatic parameter γ varies continuously with cosmic time as the universe expands. Our approach is to study how the adiabatic parameter should vary so that in the course of its evolution the universe goes a transition from an inflationary phase to a radiation-dominated phase. To describe these two early phases of the universe in a unified manner, we assume the functional form for the adiabatic parameter γ as a function of scale factor. A unified description of early evolution of universe is presented in which an inflationary phase is followed by radiation-dominated phase. The solutions show the power-law inflation. The nature of singularity and the physical properties have been discussed in detail. We adopt the convention that the signature of the metric tensor is -2 and the speed of light is taken to be unity. For notational convenience dots will denote differentiation with respect to cosmic time, commas denotes partial derivatives and semi colons denote covariant derivatives.

2 Model and Field Equations

Lau and Prokhovnik [17] proposed the generalized field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = 8\pi G T_{ij} + \psi_{,i}\psi_{,j}, \tag{1}$$

using a variation action principle. R_{ij} is the Ricci tensor, $R = R_i^i$ is the Ricci scalar, T_{ij} is the energy momentum tensor and g_{ij} is the metric tensor. The cosmological parameter Λ and gravitational constant G are functions of the scalar function ψ. The field equations (1) is a generalization of the classical Einstein's field equations and Lau's [16] theory to incorporate variables cosmological parameter Λ, gravitational parameter G. The quantity

$$\Lambda = \lambda(t) - \frac{1}{2}g^{00}\dot{\psi}^2 \tag{2}$$

is a generalization of the usual cosmological constant. Here λ is the cosmological constant used in variation of action principle by Lau and Prokhovnik.

The scalar potential ψ is strictly time dependent and couples Λ and G:

$$\psi = \psi(t), \quad \Lambda = \Lambda(\psi), \quad G = G(\psi). \tag{3}$$

In general, ψ and Λ could be functions of time and space, but since we will be considering only the spatially homogeneous Robertson–Walker models in what follows, we take them to functions of time alone. Coupled to (1), the field equation for ψ is

$$\dot{\psi}\Box\psi + \dot{\Lambda} + \frac{1}{2}\dot{g}^{00}\dot{\psi}^2 + g^{00}\dot{\psi}\ddot{\psi} + 8\pi\dot{G}L_m = 0, \tag{4}$$

where L_m is the matter Lagrangian density equal to the matter energy density including all non-gravitational fields and

$$\Box\psi = g^{ij}\psi_{;ij}. \tag{5}$$

We consider the spatially homogeneous and isotropic flat Robertson–Walker model

$$ds^2 = dt^2 - R^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{6}$$

where r, θ and ϕ are dimensionless co-moving coordinates, $R(t)$ is the scale factor governing the rate of expansion of the universe. The energy–momentum tensor can be taken as that of perfect fluid, that is,

$$T_{ij} = -pg_{ij} + (\rho + p)u_i u_j, \tag{7}$$

where ρ is the energy density and p is the pressure of fluid.

The time and space components of the field equations (1) for the metric (6) give, respectively

$$3\frac{\dot{R}^2}{R^2} - \Lambda = \dot{\psi}^2 + 8\pi G\rho, \tag{8}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \Lambda = -8\pi G p. \tag{9}$$

Using the metric (6), it is easy to show that the field equations (4) reduces to

$$2\dot{\psi}\ddot{\psi} + \dot{\Lambda} + 3\frac{\dot{R}}{R}\dot{\psi}^2 + 8\pi\dot{G}\rho = 0. \tag{10}$$

All of the field equations (1)–(10) are not independent because of the Bianchi identities. From (10), the Robertson–Walker metric (6) and the perfect fluid form of energy–momentum tensor (7), we derive the following equation on taking the divergence of (1).

$$\dot{\rho} + \rho\frac{\dot{G}}{G} + 3(\rho + p)\frac{\dot{R}}{R} + \frac{\dot{\Lambda}}{8\pi G} + \frac{1}{8\pi G}\left[(\dot{\psi^2}) + 3\frac{\dot{R}}{R}\dot{\psi}^2\right] = 0. \tag{11}$$

This reduces to the usual equation of conservation of mass–energy

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0, \tag{12}$$

if we take

$$\rho\frac{\dot{G}}{G} = -\frac{\dot{\Lambda}}{8\pi G} - \frac{1}{8\pi G}\left[(\dot{\psi^2}) + 3\frac{\dot{R}}{R}\dot{\psi}^2\right], \tag{13}$$

which also gives the result (10). Equations (8), (10) and (12) are a system of three equations for six unknowns R, ρ, p, G, Λ, and ψ. This means that we have to solve the field equations by making the assumptions for some of the unknowns.

In order to solve the system of equations (8), (10) and (12), we first assume the relation between pressure and energy density through the "gamma-law" equation of state

$$p = (\gamma - 1)\rho, \tag{14}$$

where γ is the adiabatic parameter. In general, the value of γ is taken to be constant lying between $0 \le \gamma \le 2$ for different epochs. Our aim in this paper is to study how the adiabatic parameter should vary so that in the course of its evolution the universe goes through a transition from an inflationary phase to a radiation-dominated phase. Among the many possible functional forms, Carvalho [9] assumed a scale dependent γ of the form

$$\gamma(R) = \frac{4}{3}\frac{A(R/R_*)^2 + (a/2)(R/R_*)^a}{A(R/R_*)^2 + (R/R_*)^a}, \tag{15}$$

where A is a constant, a is a free parameter and R_* is a certain reference value of scale factor R. The above functional form of γ is an increasing function of R. In the limit $R \to 0$, we have

$$\gamma(R) = \frac{2a}{3}.$$

The parameter a is related to the power of the cosmic time t during the inflationary era, and for $a \to 0$ we have an exponential inflation ($\gamma = 0$ at $R = 0$). The expression for γ in (15) expresses the evolution of the universe as it goes through a transition from an inflationary phase to a radiation-dominated phase. The function $\gamma(R)$ is such that when the scale factor R is less than a certain reference value R_* ($R \ll R_*$), we have the inflationary phase and when the scale factor is greater than R_* ($R \gg R_*$), when we have radiation-dominated phase.

Next we take two other assumptions for the system of (8)–(13). We shall fix the mathematical form of the cosmological parameter Λ and scalar field ψ by assuming

$$\Lambda = 3\alpha H^2, \tag{16}$$

$$\psi = \alpha_1 \ln R + \alpha_0, \tag{17}$$

where α, α_0 and α_1 are all positive constants and $H = \dot{R}/R$ is the Hubble parameter. The ansatz (16) initially has been proposed by Carvalho et al. [8] on the dimensional ground, widely used to study decaying vacuum cosmological models (see also, Overduin and Cooperstock [21] and Arbab [2]). The form of ψ in (17) is the simplest nonlinear one, which is compatible with the Dirac's LNH.

Using (16) and (17) in (8) and simplifying, we obtain

$$\frac{2H'}{H} = \frac{\rho'}{\rho} + \frac{G'}{G}, \tag{18}$$

where a prime denotes derivative with respect to scale factor R. Using (14), (12) can be written as

$$\frac{\rho'}{\rho} = -\frac{3\gamma(R)}{R}. \tag{19}$$

Again, using (16) and (17), (10) can be written as

$$\frac{(6\alpha + 2\alpha_1^2)}{(3 - 3\alpha - \alpha_1^2)} \frac{R}{H} H' + \frac{3\alpha_1^2}{(3 - 3\alpha - \alpha_1^2)} + \frac{G'}{G} R = 0. \tag{20}$$

Using (18) and (19) in (20), we finally obtain

$$H' + \left[\left\{1 - \alpha - (\alpha_1^2/3)\right\} \frac{3}{2}\gamma(R) + (\alpha_1^2/2)\right] \frac{H}{R} = 0, \tag{21}$$

and its integral is

$$H = D \exp\left(-\frac{3}{2}\left\{(1-\alpha) - (\alpha_1^2/3)\right\} \int \frac{\gamma(R)}{R} dR - \frac{\alpha_1^2}{2} \int \frac{1}{R} dR\right). \tag{22}$$

3 Solution of the Field Equations

Substituting (15) into (22) and integrating, we have that the Hubble parameter is given by

$$H = \frac{D}{R^{\alpha_1^2/2}[A(R/R_*)^2 + (R/R_*)^a]^{[(1-\alpha)-(\alpha_1^2/3)]}}, \tag{23}$$

where D is the constant of integration. Now we study (23) for two early phases of evolution of universe-inflationary and radiation-dominated phases.

For *inflationary phase* ($R \ll R_*$), the second term in the right-hand side of denominator of (23) dominates and one has a phase of power-law inflation which is given by

$$R = R_* \left[D\left\{\frac{\alpha_1^2}{2} + a(1-\alpha) - (a\alpha_1^2/3)\right\} t\right]^{\frac{1}{[(\alpha_1^2/2) + a(1-\alpha) - (a\alpha_1^2/3)]}}. \tag{24}$$

The Hubble parameter in terms of cosmic time t is given by

$$H = \frac{1}{[(\alpha_1^2/2) + a\{(1-\alpha) - (\alpha_1^2/3)\}]} \frac{1}{t}. \tag{25}$$

The other physical parameters have the following expressions:

$$\Lambda = \frac{3\alpha}{[(\alpha_1^2/2) + a\{(1-\alpha) - (\alpha_1^2/3)\}]^2} \frac{1}{t^2}, \tag{26}$$

$$\psi(t) = \frac{\alpha_1}{[(\alpha_1^2/2) + a\{(1-\alpha) - (\alpha_1^2/3)\}]} \ln t + \alpha_{00}. \tag{27}$$

The gravitational parameter has the form:

$$G(t) \propto t^{-b}, \tag{28}$$

where

$$b = \frac{3\alpha_1^2 - 2(\alpha_1^2 + 3\alpha)[(\alpha_1^2/2) + a\{(1-\alpha) - (\alpha_1^2/3)\}]}{[3(1-\alpha) - \alpha_1^2][(\alpha_1^2/2) + a\{(1-\alpha) - (\alpha_1^2/3)\}]}.$$

The energy density has the form

$$\rho \propto t^{-(2-b)}. \tag{29}$$

In (27), α_{00} is the constant of integration. Equation (24) shows that during the power-law inflation, the dimensions of the universe increase according to law $R \propto t^{1/[(\alpha_1^2/2)+a(1-\alpha)-(a\alpha_1^2/3)]}$ and for expanding universe we must have $[(\alpha_1^2/2) + a(1-\alpha) - (a\alpha_1^2/3)] > 0$. In addition, the constants have to be chosen so that $\ddot{R}$ is positive. Equation (24) also indicates that $R = 0$ at $t = 0$, $R \to \infty$ and $\dot{R} \to 0$ as $t \to \infty$. For physical significance $\rho > 0$, we must have the positive proportionality constant. The energy density and gravitational parameter are decreasing function of cosmic time in $0 \le b < 2$. Compatibility with Dirac LNH can be achieved if $b = 1$. The cosmological parameter varies as the inverse square of the cosmic time. This form of Λ is physically reasonable as observations suggest that Λ is very small in the present-day universe. It will approach zero when the age of the universe tends to infinity; otherwise, Λ is non-zero as a consequence of LNH. According to the recent cosmological observations (Perlmutter et al. [22], Riess et al. [24]), the universe is presently accelerating. A positive Λ causes an acceleration in the expansion of the universe, where as a negative Λ decelerates the expansion. We observe that $\dot{\psi} \propto t^{-1}$, which shows that $\dot{\psi} \to 0$ as the age of the universe increases indefinitely.

We also observe that the rate of decrease of the gravitational parameter is given by

$$\left|\frac{\dot{G}}{G}\right| = bt^{-1}. \tag{30}$$

Therefore, under most circumstances, G can be regarded as a genuine constant after a time, which is distant enough from Big Bang. The implication of a time varying G will take effect only in the early epoch. We also note that $|\dot{\Lambda}/\Lambda| = t^{-1}$. For similar reasons as in the case of G, Λ can be treated as a genuine constant after the early epoch. Hence, with the approximation of zero Λ and constant G, we can immediately go over to Einstein's field equations.

For *radiation-dominated phase* ($R \gg R_*$), we have from (23)

$$H = \frac{D}{R^{\alpha_1^2/2} A^{[(1-\alpha)-(\alpha_1^2/3)]} (R/R_*)^{2[(1-\alpha)-(\alpha_1^2/3)]}}. \tag{31}$$

Integrating (31) for $H = \dot{R}/R$ to obtain an expression for R in terms of cosmic time t, which is given by

$$R = R_* \left[D A^{[(2-2\alpha)-(\alpha_1^2/6)]} \left\{ (2-2\alpha) - (\alpha_1^2/6) \right\} t \right]^{\frac{1}{[(2-2\alpha)-(\alpha_1^2/6)]}}. \tag{32}$$

The Hubble parameter, cosmological parameter and scalar field in terms of cosmic time are respectively given by

$$H = \frac{1}{[(2-2\alpha)-(\alpha_1^2/6)]} \frac{1}{t}, \tag{33}$$

$$\Lambda = \frac{3\alpha}{[(2-2\alpha)-(\alpha_1^2/6)]^2} \frac{1}{t^2}, \tag{34}$$

$$\psi(t) = \frac{\alpha_1}{[(2-2\alpha)-(\alpha_1^2/6)]} \ln t + \alpha_{000}. \tag{35}$$

The gravitational parameter has the form:

$$G(t) \propto t^{-b_1}, \tag{36}$$

where

$$b_1 = \frac{2(36\alpha^2 - 36\alpha + \alpha_1^4 - 3\alpha_1^2 + 15\alpha\alpha_1^2)}{(3 - 3\alpha - \alpha_1^2)(12 - 12\alpha - \alpha_1^2)}.$$

The energy density has the form

$$\rho \propto t^{-(2-b_1)}. \tag{37}$$

In (35), α_{000} is the constant of integration.

The physical interpretations in the case of radiation-dominated phase are same as discussed in inflationary phase. Since $\Lambda \propto t^{-2}$, the magnitude of Λ will decrease as the age of the universe increases. For the compatibility with Dirac's LNH ($R \propto t^{1/3}$), we have

$$12\alpha = -(6 + \alpha_1^2). \tag{38}$$

An important observational quantity is the deceleration parameter $q = -R\ddot{R}/\dot{R}^2$. The deceleration parameter can be expressed as a function of the thermodynamic, gravitational constant, cosmological constant and scalar field in the form

$$q = \frac{4\pi G(t)(\rho + 3p) - \Lambda + \dot{\psi}^2/2}{8\pi G(t)\rho + \Lambda + \dot{\psi}^2}. \tag{39}$$

Current observations show that the deceleration parameter of the universe is in the range $-1 \le q < 0$, and the present day universe undergoes an accelerated expansionary evolution.

From (23), a unified expression for deceleration parameter can be given terms of scale factor as

$$q = \frac{(1 - 2\alpha - \alpha_1^2/6) A (R/R_*)^2 + [\alpha_1^2/2 + a(1 - \alpha - \alpha_1^2/3) - 1](R/R_*)^a}{[A(R/R_*)^2 + (R/R_*)^a]}. \tag{40}$$

Therefore, q varies from $[(\alpha_1^2/2)+a(1-\alpha)+(a\alpha_1^2/3)-1]$ for $R \ll R_*$ to $[(1-2\alpha)-(\alpha_1^2/6)]$ for radiation phase. The sign of the deceleration parameter indicates whether the model inflates or not. The negative value of q accelerates the universe where as positive value decelerates the universe.

We now study the solution in the limit $a \to 0$. In this case, (23) becomes

$$H = \frac{H_I}{R^{\alpha_1^2/2}[A(R/R_*)^2+1]^{[(1-\alpha)-(\alpha_1^2/3)]}}, \tag{41}$$

where H_I is a constant. In the limit of very small R, the second term on the right hand side in denominator of (41) dominates and one has power-law inflation, which is given by

$$R = \left[\frac{H_I\alpha_1^2}{2}t\right]^{2/\alpha_1^2}. \tag{42}$$

Again, the radiation phase is described by the same solution as obtained in (32). We observe that for $a=0$, the universe is finitely old, since $R \to 0$ as $t \to 0$. There is the singularity since the energy density assumes infinite value as $R \to 0$. It can be inferred from (8) and (41) that for $R = 0$, $\rho \to \infty$.

The deceleration parameter has the following form in terms of R

$$q = \frac{[2\{(1-\alpha)-(\alpha_1^2/3)\}-(1-\alpha_1^2/2)]A(R/R_*)^2-1}{A(R/R_*)^2+1}, \tag{43}$$

and varies from $q=-1$ at $R=0$ to $q=[(1-2\alpha)-(\alpha_1^2/6)]$ for $R \gg R_*$ as expected.

4 Conclusion

In this paper we have discussed the flat FRW model filled with perfect fluid in scalar-tensor theory proposed by Lau and Prokhovnik. We have solved the generalized field equations for two early phases of evolution of universe by using variation of "gamma-law" equation of state, in which the adiabatic parameter 'gamma' varies continuously as universe expands. For $a=0$, the parameter γ varies from 0 for $R=0$ to $4/3$ when $R \gg R_*$. The universe is finitely old and has power-law inflation. There is a big bang singularity since the energy density diverges as $R \to 0$, which gives a similar result as obtained by Carvalho in General Relativity Theory. We have assumed the cosmological parameter is a quadratic function of the Hubble parameter and that the scalar field is a logarithmic function of the scale factor.

For a in the range $0 < a < 1$, γ slowly increases from $2a/3$ for inflationary phase to radiation-dominated phase $(4/3)$. The first period of evolution is described by inflationary phase with power-law inflation. This is then followed by a radiation-dominated era, again with power-law expansion. The cosmological parameter is a decreasing function of time, while the gravitational parameter tends to a constant value at late times. The implication of time varying G will take effect only in the early epoch. The possibility that the cosmological parameter and gravitational parameter are not real constant is an intriguing possibility, which has intensively been investigated in the physical literature. It is a very plausible hypothesis that these effects were much stronger in the early universe. The Λ-term varies inversely as square of the cosmic time which is consistent with the observation of the present-day values of the cosmological constant which are very small. The energy density and gravitational constant decrease as the age of the universe increases. The universe starts with zero volume and expands so ever with cosmic time. The possibility of a positive cosmological parameter and negative deceleration parameter emerged in our results, which indicate that the present universe is accelerating. The results can be made compatible with Dirac's LNH. The results show a generalization of Carvalho's work to a simple scalar-tensor theory. Thus, we have showed that by using a varying adiabatic parameter, it is possible to describe the two early phases of evolution of the universe in a unified manner. The solutions obtained in the present paper could give an appropriate description of the early period of our universe as expected. As future work, we are studying whether a variable gamma-law equation of state could explain the continuous transition from a decelerated universe to an accelerated one, as is currently observed.

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